GCE A LEVEL

1305U50-1

S23-1305U50-1

WEDNESDAY, 14 JUNE 2023 – AFTERNOON

FURTHER MATHEMATICS – A2 unit 5
FURTHER STATISTICS B

1 hour 45 minutes

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 80.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Additional Formulae for 2023

Laws of Logarithms

$$\log_a x + \log_a y \equiv \log_a (xy)$$
$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$$
$$k \log_a x \equiv \log_a \left(x^k\right)$$

Sequences

General term of an arithmetic progression:

$$u_n = a + (n-1)d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

Mensuration

For a circle of radius, r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Calculus and Differential Equations

Differentiation

Function	Derivative
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f\left(g(x)\right)$	$f'\left(g(x)\right)g'(x)$

Integration

Function	Integral
$f'\left(g(x)\right)g'(x)$	$f\left(g(x)\right) + c$

Area under a curve $= \int_a^b y \, \mathrm{d}x$

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. The average time it takes for a new kettle to boil, when full of water, is 305 seconds. An old kettle will take longer, on average, to boil. Alun suspects that a particular kettle is an old kettle. He boils the full kettle on 9 occasions and the times taken, in seconds, are shown below.

305 295 310 310 315 307 300 311 306

You may assume the times taken to boil the full kettle are normally distributed.

(a) Calculate unbiased estimates for the mean and variance of the times taken to boil the full kettle. [3]

(b) Test, at the 5% level of significance, whether there is evidence to suggest that this is an old kettle. [7]

(c) State a factor that Alun should control when carrying out this investigation. [1]

1305U501 03

2. The random variables X and Y are independent, with X having mean μ and variance σ^2, and Y having mean μ and variance $k\sigma^2$, where k is a positive constant.

Let $\overline{X}$ denote the mean of a random sample of 20 observations of X, and let $\overline{Y}$ denote the mean of a random sample of 25 observations of Y.

(a) Given that $T_1 = \dfrac{3\overline{X} + 7\overline{Y}}{10}$, show that T_1 is an unbiased estimator for μ. [2]

(b) Given that $T_2 = \dfrac{\overline{X} + a^2\overline{Y}}{1 + a}$, $a > 0$, and T_2 is an unbiased estimator for μ, prove that $a = 1$. [3]

(c) Find and simplify expressions for the variances of T_1 and T_2. [5]

(d) Show that the value of k for which T_1 and T_2 are equally good estimators is $\dfrac{5}{6}$. [3]

(e) Given that $T_3 = (1 - \lambda)\overline{X} + \lambda\overline{Y}$, find an expression for λ, in terms of k, for which T_3 has the smallest possible variance. [6]

TURN OVER

3. Athletes who compete in the 400 m event have resting heart rates (RHR), measured in beats per minute, which are normally distributed with known standard deviation 4·7. A random sample of 90 athletes who compete in the 400 m event is taken. Their resting heart rates are summarised by

$$\sum x = 4014 \qquad \text{and} \qquad \sum x^2 = 182257.$$

(a) Find a 99% confidence interval for the mean of the RHR of athletes who compete in the 400 m event. Give the limits of your interval correct to 1 decimal place. [5]

(b) Without doing any further calculation, explain how the width of a 95% confidence interval would compare to the width of your interval in part (a). [1]

Athletes who compete in the discus event have RHR which are normally distributed with known standard deviation σ. A random sample of 100 athletes who compete in the discus event is taken. A 95% confidence interval for the mean of the RHR is calculated as [49·4, 52·6].

(c) Determine the value of σ that was used to calculate this confidence interval. [3]

(d) Referring to the confidence intervals, state, with a reason, what can be said about the RHR of athletes who compete in the 400 m event compared to the RHR of athletes who compete in the discus event. [2]

4. Llŷr believes that he will have more social media followers by appearing on a certain Welsh television show. To investigate his belief, he collects data on 9 randomly selected contestants who have appeared on the show. Llŷr records the number of social media followers one week before and one week after the contestants appeared on the show. The data he collects are shown in the table below.

Contestant	A	B	C	D	E	F	G	H	I
Before	480	1008	0	344	351	781	876	741	457
After	841	998	751	344	954	542	820	1011	644

(a) (i) Carry out a Wilcoxon signed-rank test on this data set, at a significance level as close to 10% as possible. [9]

(ii) Suggest a possible course of action that Llŷr might take. [1]

(b) Give two reasons why the Wilcoxon signed-rank test is appropriate in this case. [2]

5. The masses, X, in kg, of men who work for a large company are normally distributed with mean 75 and standard deviation 10.

(a) Find the probability that the mean mass of a random sample of 5 men is less than 70 kg. [3]

(b) The mean mass, in kg, of a random sample of n men drawn from this distribution is $\bar{X}$. Given that P$\left(\bar{X} > 80\right)$ is approximately 0·007, find n. [5]

The masses, in kg, of women who work for the company are normally distributed with mean 68 and standard deviation 6. A lift in the company building will not move if the total mass in the lift is more than 500 kg.

(c) A random sample of 3 men and 4 women get in the lift. Find the probability that the lift will not move. [4]

(d) State a modelling assumption you have made in calculating your answer for part (c). [1]

6. A triathlon race organiser wishes to know whether competitors who are members of a triathlon club race more frequently than competitors who are not members of a triathlon club. Six competitors from a triathlon club and six competitors who are not members of a triathlon club are selected at random. The table below shows the number of triathlon races they each entered last year.

Club members	1	14	12	5	3	7
Not club members	2	9	4	0	8	6

(a) Use a Mann-Whitney U test at a significance level as close to 5% as possible to carry out the race organiser's investigation. [6]

(b) Briefly explain why a Wilcoxon signed-rank test is not appropriate in this case. [1]

TURN OVER

7. Branwen intends to buy a new bike, either a *Cannotrek* or a *Bianchondale*. If there is evidence that the difference in the mean times on the two bikes over a 10 km time trial is more than 1·25 minutes, she will buy the faster bike. Otherwise, she will base her decision on other factors.

She negotiates a test period to try both bikes. The times, in minutes, taken by Branwen to complete a 10 km time trial on the *Cannotrek* may be modelled by a normal distribution with mean μ_C and standard deviation 0·75.

The times, in minutes, taken by Branwen to complete a 10 km time trial on the *Bianchondale* may be modelled by a normal distribution with mean μ_B and standard deviation 0·6.

During the test period, she completes 6 time trials with a mean time of 19·5 minutes on the *Cannotrek*, and 5 time trials with a mean time of 17·3 minutes on the *Bianchondale*.

She calculates a p% confidence interval for $\mu_C - \mu_B$.

(a) What would be the largest value of p that would lead Branwen to base her purchasing decision on the time trials, without considering other factors? [6]

(b) State an assumption you have made in part (a). [1]

END OF PAPER

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